Inequivalent classes of closed three-level systems

Andrei B. Klimov
Departamento de Física, Universidad de Guadalajara, Revolución 1500, 44420 Guadalajara, Jalisco, Mexico

Hubert de Guise
Department of Physics, Lakehead University, Thunder Bay, Ontario, Canada P7B 5E1

Luis L. Sánchez-Soto
Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain
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We show here that the Λ and V configurations of three-level atomic systems, while they have recently been shown to be equivalent for many important physical quantities when driven with classical fields [M. B. Plenio, Phys. Rev. A **62**, 015802 (2000)], are no longer equivalent when coupled via a quantum field. We analyze the physical origin of such behavior and show how the equivalence between these two configurations emerges in the semiclassical limit.

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Atomic coherence is essential to properly understand the response of an atomic three-level system to laser radiation (for a recent review see Ref. [1]). A large amount of research has thus been devoted to exploring many effects that rely on quantum interference in atomic systems: examples include dark states [2], narrow spectral lines [3], pulse matching [4], and antiintuitive excitation [5]. These nonclassical features have an enormous variety of interesting and nontrivial consequences, including electromagnetically induced transparency [6], lasing without inversion [7], state-selective molecular excitation [8], and demonstrations of slow light [9] and fast light [10], to mention only a few examples.

Roughly speaking, one can identify dark states as a key concept in the description of these coherent phenomena: a dark state is a specific coherent superposition resulting, by destructive quantum interference, in a completely decoupled state. So, the atom prepared in a dark state cannot be excited and cannot leave the dark state.

When we limit the discussion to the case in which only two transitions are allowed between levels, there are three distinct level configurations known as Ξ, Λ, and V [11]. It is well known that dark states cannot be formed in the Ξ configuration: this is the reason why quantum interference does not play any role for this system and it is tacitly considered inequivalent to Λ and V configurations.

Usually the phenomenon of spontaneous emission (which is the main damping contribution) plays a destructive role in the creation of this coherence. Note, however, that there have been several proposals in which coherence induced from the spontaneous emission itself is used for the preparation of the atom [12].

One can find, scattered in the literature, statements about similarities between Λ- and V-type systems in some limits or under different decaying rates between levels [13]. The recent and intriguing paper by Plenio [14] sheds light on these similarities, pointing out a more general equivalence between these systems. The central result is that both schemes share a common structure and, as a consequence, exhibit the same physical behavior for many important quantities. In general, to derive this equivalence, one writes the master equations of both systems and, by a smart change of variables, shows that these equations are identical.

Although such equivalence between Λ and V schemes is valid for many purposes, it is always deduced in a regime where the fields are essentially classical. It remains to investigate the extent to which these systems remain equivalent when they interact with quantum fields. It is precisely the objective of this paper to answer this question.

We begin by considering a collection of A identical three-level atoms confined to a small volume with linear dimensions less than the relevant wavelengths of light. The atomic energy levels are always ordered accordingly $E_1 \leqslant E_2 \leqslant E_3$. The collective atomic operators are denoted by S_{ij} (the Latin indices run from 1 to 3) and satisfy the commutation relations

$$[S_{ij}, S_{kl}] = \delta_{jk} S_{il} - \delta_{il} S_{kj}, \tag{1}$$

distinctive of the algebra u(3). For concreteness, we shall treat only fully symmetrical states; then, S_{ij} is conveniently realized, in the second quantization formalism, by boson operators

$$S_{ij} = b_i^\dagger b_j, \tag{2}$$

which transfer excitations from level j to level i $(i \neq j)$. The eigenvalue of S_{ii} is just the population of level i.

The atomic system interacts with a single quantum field of frequency ω described by the usual creation and destruction operators $a^\dagger$ and a, respectively (the case of two-mode fields can be treated much in the same way). The general form of the Hamiltonian for our systems is $H = H_0 + H_{\mathrm{int}}$, where the free Hamiltonian is (in units $\hbar = 1$)

$$H_0 = \sum_{i=1}^{3} E_i S_{ii} + \omega a^\dagger a, \tag{3}$$

and the interaction Hamiltonian depends on the level configurations,

$$H_{\rm int}^{(\Lambda)}=g_{31}(X_{31}+X_{31}^{\dagger})+g_{32}(X_{32}+X_{32}^{\dagger}),$$

$$H_{\rm int}^{(V)}=g_{31}(X_{31}+X_{31}^{\dagger})+g_{21}(X_{21}+X_{21}^{\dagger}), \quad (4)$$

which have been written in terms of operators in the so-called su(3) deformed algebra [15]

$$X_{31}=aS_{31}, \quad X_{21}=aS_{21}, \quad X_{32}=aS_{32}, \quad (5)$$

and $X_{ij}=(X_{ji})^{\dagger}$. These first-order transition operators describe allowed (direct) transitions between the corresponding atomic levels, accompanied by the appropriate emission or absorption of a photon.

Note that a pair of levels (lower levels, in the case of Λ, upper levels, in the case of V) must be nearly degenerate in order to interact efficiently with a single field mode. In the case of degenerate levels for the Λ system, we write $E_1=E_2\equiv E_-$, $E_3\equiv E_+$, and rotate the operators b_j, which enter in the representation of atomic operators Eq. (2), to

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}=\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad b_3=c_3, \quad (6)$$

where c_1 and c_2 are new destruction operators and $\tan\alpha=g_{32}/g_{31}$. In terms of new atomic operators $\tilde{S}_{jk}=c_j^{\dagger}c_k$ the transformed Hamiltonian becomes

$$\tilde{H}^{(\Lambda)}=h^{(\Lambda)}+E_-\tilde{S}_{22},$$

$$h^{(\Lambda)}=\omega a^{\dagger}a+E_+\tilde{S}_{33}+E_-\tilde{S}_{11}+g_{\Lambda}(a\tilde{S}_{31}+a^{\dagger}\tilde{S}_{13}), \quad (7)$$

where $g_{\Lambda}=g_{31}\cos\alpha+g_{32}\sin\alpha$.

The dynamics of the uncoupled level $|\tilde{2}\rangle=-\sin\alpha|1\rangle+\cos\alpha|2\rangle$ is completely independent of the field variables and is governed by the sub-Hamiltonian $E_-\tilde{S}_{22}$ of $\tilde{H}^{(\Lambda)}$. The levels $|\tilde{1}\rangle$ and $|\tilde{3}\rangle$ are coupled via an effective coupling constant g_{Λ}.

The same procedure can be repeated for $H^{(V)}$, rotating this time b_2 and b_3. Using $E_1=E_-$ and $E_2=E_3\equiv E_+$ we obtain

$$\tilde{H}^{(V)}=h^{(V)}+E_+\tilde{S}_{22},$$

$$h^{(V)}=\omega a^{\dagger}a+E_-\tilde{S}_{11}+E_+\tilde{S}_{33}+g_V(a\tilde{S}_{31}+a^{\dagger}\tilde{S}_{13}), \quad (8)$$

where $g_V=g_{21}\sin\beta+g_{31}\cos\beta$ and $\tan\beta=g_{21}/g_{32}$. A simple look at the transformed Hamiltonians (7) and (8) immediately shows that they both have dark states and the dynamics of the remaining two-level subsystems is the same. This is the basis on which rests the claim of dynamical equivalence between Λ and V configurations.

On closer examination, the complete equivalence between Λ and V configurations should also include higher-order processes, since the action of first-order operators defined in Eq. (5) is intrinsically nonlinear [15]. In what follows, we concentrate on second-order feasible processes like the one represented by $X_{23}X_{31}=a^{\dagger}a(S_{33}+1)S_{21}$ (for the Λ scheme). This results in a net transfer of one atomic excitation between the degenerate levels from $|1\rangle$ to $|2\rangle$, with the transition enhanced by a factor which depends both on field and atomic populations. This enhancement is different for the process $X_{31}X_{23}=(a^{\dagger}a+1)S_{33}S_{21}$, which also results in a transfer from $|1\rangle$ to $|2\rangle$, so that the second-order operator

$$[X_{31},X_{23}]=(S_{33}-a^{\dagger}a)S_{21} \quad (9)$$

describes an effective intensity-dependent transition $1\leftrightarrow 2$, stimulated by the field strength and the population in the intermediate level. In particular, the commutator will vanish if the population of the intermediate state is precisely equal to the total number of photons in the system.

Take now the second-order operator for the V scheme,

$$[X_{31},X_{12}]=(S_{11}+a^{\dagger}a+1)S_{32}, \quad (10)$$

which measures the difference between two two-step transfers of excitation between the degenerate upper atomic levels. Again, this second-order operator depends on the population of the intermediate state and on the photon population. However, this commutator will never vanish.

To graphically illustrate the differences between both schemes, we shall use a rootlike diagram constructed in the following way. We choose a Cartan subalgebra (i.e., maximal set of commuting operators) containing the two independent inversions $h_1=S_{11}-S_{22}$ and $h_2=S_{22}-S_{33}$ for Λ and $h_1=S_{22}-S_{11}$ and $h_2=S_{33}-S_{22}$ for V. Then, we define the weight components κ_1 and κ_2 through the "eigenvalue" equations for first-order operators in Eq. (5),

$$[h_1,X_{ij}]=\kappa_1 X_{ij}, \quad [h_2,X_{ij}]=\kappa_2 X_{ij}, \quad (11)$$

and analogously for second-order operators. The eigenvalues (κ_1,κ_2) obtained for relevant first- and second-order operators are then placed on a two-dimensional diagram, using as basis the vectors of the su(3) root diagram, which are angled at $2\pi/3$ to one another. One then draws from the center vectors to the points on the diagram. This is illustrated in Fig. 1.

We recall that the major feature of this weight diagram is that commutation is mapped, up to a multiplicative factor, to vector addition [16]. For instance, the result of $[X_{31},X_{23}]$ is proportional to the vector resulting from the addition of the root vectors for X_{31} and X_{23}.

It is clear by inspection of Eqs. (9) and (10), and from Fig. 1, that Λ and V configurations are not equivalent when coupled via a quantum field: no unitary transformation acting on the atomic operators can transform $H^{(\Lambda)}$ into $H^{(V)}$. In particular, no relabeling of the atomic states transforms a Λ into a V: the structure of the second-order operators prevents this.

As one would expect, all differences vanish in systems where first-order operators contain classical rather than quantum fields. If a is replaced by a complex number α, the transition operators of both schemes reduce to su(3) operators and close on equivalent su(3) algebras: in the Λ case, the first-order operators become $\alpha S_{32}, \alpha S_{31}$ and their conjugates, while second-order operators $[\alpha S_{31},\alpha^* S_{23}]$ reduces to $-\alpha^*\alpha S_{21}$. A similar argument applies to the V case. The equivalence found by Plenio can then simply be expressed

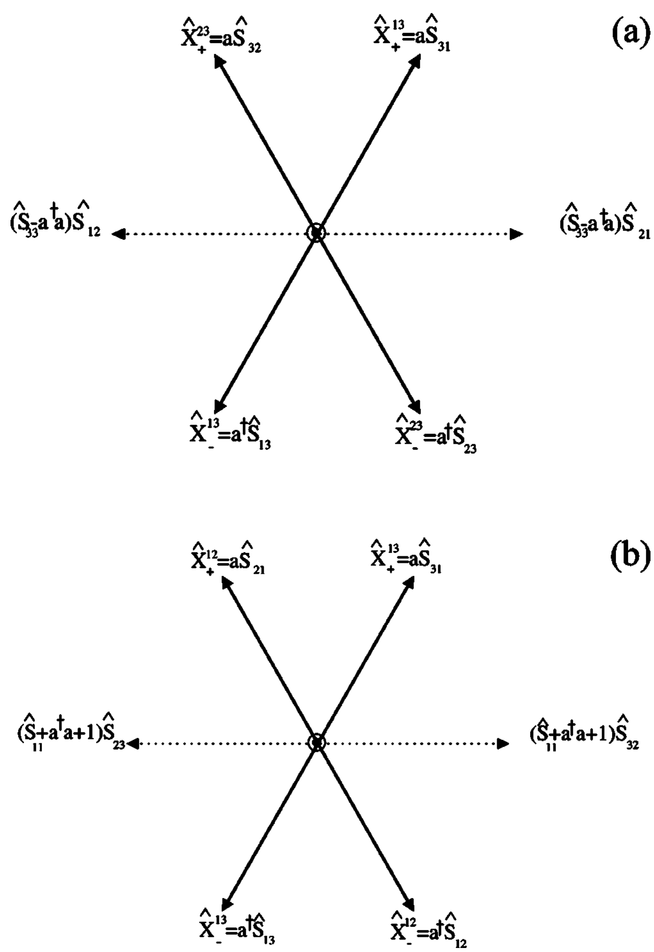

FIG. 1. A rootlike diagram for the first-order operators (thick lines) and second-order operators (dashed lines) for (a) Λ and (b) V schemes.

by the statement that the first-order operators for Λ can be transformed into the first-order operators for V by geometrical reflection of the root vector; it is this reflection which effects the relabeling of basis states proposed in Ref. [14].

Although differences will certainly be noticeable when the number of field quanta and the level population are both low, we observe that these differences can be important even in strong fields when the number of atoms A is large.

As a simple though remarkable application of the above discussion, we consider the dynamics of the Λ and V configurations in the dispersive regime, when

$$|\Delta_{ij}| \gg A g_{ij}\sqrt{\langle a^{\dagger}a\rangle+1}, \tag{12}$$

with $\Delta_{ij}=E_i-E_j-\omega$. Following Ref. [17], let us define the following unitary transformations:

$$U(\varepsilon_{ij})=\exp[\varepsilon_{ij}(X_{ij}-X_{ij}^{\dagger})], \tag{13}$$

where

$$\varepsilon_{ij}=\frac{g_{ij}}{\Delta_{ij}} \ll 1 \tag{14}$$

are small parameters in this regime. One then shows that $\tilde{H}_{\rm eff}^{(\Lambda)}=U(\varepsilon_{32})U(\varepsilon_{31})H^{(\Lambda)}U^{\dagger}(\varepsilon_{31})U^{\dagger}(\varepsilon_{32})$ is the effective Hamiltonian,

$$\tilde{H}_{\rm eff}^{(\Lambda)}=\varepsilon_{31}g_{32}(S_{12}+S_{12}^{\dagger})(S_{33}-a^{\dagger}a), \tag{15}$$

where we have omitted diagonal terms that contain the dynamical Stark shift [18]. It is clear that there will be no population transfer between levels $|1\rangle$ and $|2\rangle$ when the population of $|3\rangle$ is exactly equal to the number of field quanta. In particular, there will be no transfer if the field is in the vacuum and level $|3\rangle$ is unoccupied.

Applying the same method to $H^{(V)}$, we get $\tilde{H}_{\rm eff}^{(V)}=U(\varepsilon_{31})U(\varepsilon_{21})H^{(V)}U^{\dagger}(\varepsilon_{21})U^{\dagger}(\varepsilon_{31})$, where

$$\tilde{H}_{\rm eff}^{(V)}=\varepsilon_{21}g_{13}(S_{32}+S_{32}^{\dagger})(S_{11}+a^{\dagger}a+1). \tag{16}$$

In contrast with the results for the Λ configuration, there is *always* a population transfer between the degenerate levels $|2\rangle$ and $|3\rangle$ via the intermediate level in the V configuration. The transfer of excitations between levels $|2\rangle$ and $|3\rangle$ in the V configuration takes place even when level $|2\rangle$ is unpopulated and there are no field quanta. This occurs because of the spontaneous emission (stimulated by the zero-point fluctuations of the quantum field) from $|2\rangle$ to $|1\rangle$ with a subsequent absorption of the emitted photon leading to the population of the upper levels. Once again, these differences disappear in the limit of classical fields.

In conclusion, we have shown that the Λ and V configurations cannot be taken as equivalent if we treat the photon field as a quantized field. It is also possible to see why, physically, these configurations are different: in a V configuration, vacuum fluctuations can create a photon, the absorption of which acts as a trigger for the transfer of excitation between levels $|2\rangle$ and $|3\rangle$. This transfer mechanism cannot occur in the Λ configuration.

It is the hope that these basic results will help to elucidate the origin of equivalences between different three-level schemes, also when extra decay rates for the levels are taken into account.

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