GEOMETRIC PHASE IN SU(N) INTERFEROMETRY

Hubert de Guise*

C.R.M., Université de Montréal, C.P. 6128-A, Montréal, H3C 3J7, Canada

Barry C. Sanders, Stephen D. Bartlett and Weiping Zhang

Department of Physics and Center for Lasers and its applications, Macquarie University, Sydney, N.S.W. 2109, Australia

Abstract

An interferometric scheme to study Abelian geometric phase shift over the manifold SU(N)/SU(N-1) is presented.

I. INTRODUCTION

The purpose of this contribution is twofold: to review how an SU(N) transformation can be experimentally realized using optical elements, and to show how such an experimental realization can be used to investigate the cyclic evolution of a state over the manifold SU(N)/U(N-1). The bulk of the results will be presented explicitly for SU(3) (see [1] for further details) and SU(4), although it will become clear that the method can be applied to any SU(N).

Recall that cyclic evolution of a wave function yields the original state plus a phase shift, and this phase shift is a sum of a dynamical phase φ_d and a geometric (or topological, or quantal, or Berry) phase φ_g shift [2,3]. The geometric phase shift is important not just for quantum systems but also for all of wave physics. Thus far, controlled geometric–phase

*now at Faculté Saint-Jean, University of Alberta, 8406 rue Marie-Anne Gaboury, Edmonton, T6C 4G9, Canada

experiments, both realized and proposed, have been exclusively concerned with the abelian geometric phase arising in the evolution of U(1)–invariant states [2,4–6]

Here, we generalize the above results to an Abelian geometric phase which arises from geodesic transformations of U(N-1)–invariant states in SU(N)/U(N-1) space. The scheme employs a sequence of optical element, henceforth called SU(N) elements because they perform transformations described by an SU(N) matrix, arranged so that the net result of the sequence cyclically evolves an initial state back to itself up to a phase. It will be seen that the decomposition of an SU(N) transformation into a product of appropriate SU(2) subgroup transformations is the prescription to construct each SU(N) element as a sequence of SU(2) elements.

It is important to distinguish the evolution of states in the geometric space SU(N)/U(N-1) from the transformations of the optical beam as it progresses through the interferometer. It is possible to set up the experiment so as to eliminate the dynamical phase associated with these optical transformations, thus making the dynamical phase irrelevant for our purpose. The cyclic evolution described here occurs in the geometric space, and the geometric phase of interest is related to this evolution.

II. SU(N) OPTICAL ELEMENTS

Consider an optical element which mixes two input beams. It is, formally, a black box which performs some transformation, as the output is not the same as the input. We are here interested by optical elements which mix the input beams in a linear way, *i.e.* the output is a linear combination of the inputs. Furthermore, we will assume that the optical element is passive, *i.e.* it does not globally create or annihilate photons.

The optical elements the enter in the construction of SU(N) device are beam splitters, mirrors and phase shifters. A phase shifter is essentially a slab of material which increases the optical path lenght of one beam relative to the other. A beam splitter is a partially–silvered mirror which lets photons through with some probability.

Provided that losses can be ignored, each of these optical elements can be associated with an SU(2) unitary transformation [7,8]. It is therefore advantageous to factorize each SU(N) transformation into a product of $SU(2)_{ij}$ subgroup transformations mixing fields i and j.

An optical element mixing two fields is associated with an SU(2) transformation in the following way. Suppose that *one* photon enters the black box. We may assume that it will enter the optical system either via beam one or beam two. Thus the Hilbert space of input states is two–dimensional. As the transformation is linear, the set of all possible output states will also be a two–dimensional space. Clearly this conclusion does not change if the input state is a general state $(\alpha, \beta)^t$, where the photon has probability $|\alpha|^2$ of being in beam 1 and probability $|\beta|^2$ of being in beam 2 (α, β are complex numbers).

Suppose now that *two* photons enter the black box. Then, we can have one of three possibilities: two photons enter in beam 1, one photon enters in each beam, or both photons enter in beam 2. In this case, the Hilbert space of states is three–dimensional.

Continuing in this way, and using the fact that the input photons are indistinguishable, one rapidly works out that, in a system containing λ photons, the relevant Hilbert space is of dimension $\lambda + 1$.

The conservation of photon number leads to the following constraint on the possible form of the linear transformation. Consider first the case of a single photon. The optical transformation

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} , \tag{1}$$

with a, b, c, d complex numbers, transforms a general state $(\alpha_{in}, \beta_{in})^t$ into the output state $(\alpha_{out}, \beta_{out})^t$ such that

$$\begin{pmatrix} \alpha_{out} \\ \beta_{out} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_{in} \\ \beta_{in} \end{pmatrix} . \tag{2}$$

Taking the transpose complex conjugate of that to find $(\alpha^*_{out}, \beta^*_{out})$, multiplying from the right by $(\alpha_{out}, \beta_{out})^t$, we find that, if the number of photon (=1) is to be conserved for any input state, $|\alpha_{out}|^2 + |\beta_{out}|^2 = |\alpha_{in}|^2 + |\beta_{in}|^2 = 1$ implies that $T^\dagger \cdot T = 1$, the unit matrix. Thus, T is a 2×2 unitary transformation. Because we are only interested in the relative phase between the beams, the determinant of T can chosen without loss generality to be $+1$, so that T is an SU(2) matrix.

If the black box performs a transformation T that is an SU(2) transformation when there is a single photon in the system, it must also perform an SU(2) transformation when there are $\lambda + 1$ photons in the system: the transformtation effected by the black box cannot depend on the number of photons in the system (at least not in the regimes that we are considering). Thus, in a system of two photons, where state space is three–dimensional, T will be 3×3 representation of the relevant SU(2) transformation. In a system containing λ photon, T will be an SU(2) matrix of dimension $(\lambda + 1) \times (\lambda + 1)$ [7,8].

It is well known that an SU(2) transformation can be factored into a product of three subtransformations:

$$\begin{matrix} R_z(\alpha) & \cdot & R_y(\beta) & \cdot & R_z(\gamma) \\ \begin{pmatrix} e^{i\alpha} & \\ & e^{-i\alpha} \end{pmatrix} & & \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} & & \begin{pmatrix} e^{i\gamma} & \\ & e^{-i\gamma} \end{pmatrix} \end{matrix} \tag{3}$$

This factorization is a prescription on how to construct the SU(2) device: a slab of material is inserted in one beamline so as to create a relative phase shift of $e^{2i\gamma}$, a partially silvered mirror which lets $\cos^2\beta$ photons from beam 1 through is then inserted, and another phase shifter completes the design.

In an SU(3) interferometer, an general SU(3) matrix is decomposed into a product of three SU(2) matrices [9]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & a_1 & b_1 \\ 0 & -b_1^* & a_1^* \end{pmatrix} \cdot \begin{pmatrix} e^{i\alpha}\cos t & -\sin t & 0 \\ \sin t & e^{-i\alpha}\cos t & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & a_2 & b_2 \\ 0 & -b_2^* & a_2^* \end{pmatrix}, \tag{4}$$

where $|a_i|^2 + |b_i|^2 = 1$.

This factorization, symbolically written $R_{23}^1 \cdot R_{12} \cdot R_{23}^2$, is a *de facto* prescription on how to build the SU(3) device: fields 2 and 3 are mixed followed by a mixing of the output field 2 with the field in channel 1, and, finally, the output field 2 is mixed with field 3.

As it is possible to factorize an SU(N) matrix in terms of SU(2) submatrices [10], the process of constructing a general SU(N) device is perfectly obvious and follows the lines illustrated explicitly for the SU(3) device.

For instance, the appropriate factorization of an SU(4) matrix is a product of the type

$$R_{23}^1 \cdot R_{34}^1 \cdot R_{23}^2 \cdot R_{12} \cdot R_{23}^3 \cdot R_{34}^2 \cdot R_{23}^4 \,, \tag{5}$$

where $R_{k\ell}^i$ is an SU(2) matrix mixing fields k and ℓ.

III. GEODESIC EVOLUTIONS

The total phase φ acquired by a state during a generic cyclic evolution is the sum of $\varphi_d + \varphi_g$.

A special type of evolution is the geodesic evolution [11]; by transforming the output state along geodesic paths in the geometric space, the geometric phase shift along each path is zero.

An essential property of geodesic evolutions is that they are not transitive: the product of two such evolutions is not necessary another geodesic evolution. This is most easily illustrated by drawing three points on a plane at random. It is well known that the geodesic on a plane is a straight line. Let $|1\rangle\,, |2\rangle$ and $|3\rangle$ denote the three points. Then it is clear that, even if R_{12} is the straight (geodesic) line that connects $|1\rangle$ and $|2\rangle$, and even if R_{23} is the straight line that connect $|2\rangle$ and $|3\rangle$, the combined segment $R_{23} \cdot R_{12}$ is *not* a geodesic between $|1\rangle$ and $|3\rangle$.

This property makes it possible to construct a cyclic evolution from a sequence of geodesic legs: the geometric phase acquired during the circuit is then a global property of the entire circuit.

For definiteness, let us consider SU(3). There, the evolution of the state $\psi^{(1)}$ to the state $\psi^{(4)} = e^{i\varphi_g}\psi^{(1)}$ via 3 geodesic paths in the geometric space can be described by 3 one–parameter SU(3) group elements $\{U_k^g(s_k);\ k=1,2,3\}$, with s_k an evolution parameter. These transformations satisfy the conditions that $U_k^g(0)$ is the identity element and

$$U_k^g(s_k^0)\psi^{(k)} = \psi^{(k+1)}, \quad k=1,2,3\,, \tag{6}$$

for some fixed end values $\{s_k^0\}$ of the evolution parameters. We consider evolutions $U_k^g(s_k)$ of the form

$$U_k^g(s_k) = V_k \cdot R_{s_k} \cdot V_k^{-1}\,, \tag{7}$$

with V_k an element of SU(3) satisfying $\langle\psi^{(k)}|U_k^g(s_k)|\psi^{(k)}\rangle$ real and positive, and

$$R_{s_k} \equiv \begin{pmatrix} \cos s_k & -\sin s_k & 0 \\ \sin s_k & \cos s_k & 0 \\ 0 & 0 & 1 \end{pmatrix}\,. \tag{8}$$

The form of the one–parameter subgroup R_{s_k} with real entries was guided by the definition of a geodesic curve between two states $\psi^{(k)}$ and $\psi^{(k+1)}$, which can be written in the form [12]

$$\psi(s_k) = \psi^{(k)}\cos s_k + \frac{\left(\psi^{(k+1)} - \psi^{(k)}\langle\psi^{(k+1)}|\psi^{(k)}\rangle\right)}{\sqrt{1-\langle\psi^{(k+1)}|\psi^{(k)}\rangle^2}}\sin s_k\,, \tag{9}$$

with $0 \le s_k \le s_k^0 = \arccos\langle\psi^{(k+1)}|\psi^{(k)}\rangle$. As it is always possible to choose unit vectors $\psi^{(k)}$ such that $\langle\psi^{(k+1)}|\psi^{(k)}\rangle$ is real and positive, it is straightforward to show that any $U_k^g(s_k)$ of the form given by Eq. (7) satisfying $\langle\psi^{(k+1)}|\psi^{(k)}\rangle$ real and positive gives evolution along a geodesic curve in SU(3)/U(2).

The form of the geodesic evolution makes it easy to obtain its physical interpretation. The transformation R_{s_k} is a transformation of appropriate length along some reference geodesic (some generalized Greenwich meridian on SU(3)/U(2)). The transformation V_k is a principal axis transformation which correctly orients the reference geodesic so that it passes through $|\psi^k\rangle$ and $|\psi^{k+1}\rangle$. V_k therefore depends on the initial and final states.

The three states in SU(3)/U(2) must be chosen in a sufficiently general way to ensure that they can represent any triangle in SU(3)/U(2) [12]. Since the latter is of dimension 4, there are 4 free parameters to be chosen. The first state can be chosen, WLOG, to be the "north pole" state. Again WLOG, the second state can always be chosen to lie along the reference geodesic some distance away from the initial state. The last state must therefore contain the remaining 3 parameters. In short, the vertices of a geodesic triangle in SU(3)/U(2) can, in general, be chosen as

$$\psi^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = e^{-i\varphi_g}\psi^{(4)}, \quad \psi^{(2)} = \begin{pmatrix} \cos s_1^0 \\ \sin s_1^0 \\ 0 \end{pmatrix},$$

$$\psi^{(3)} = \begin{pmatrix} \cos s_1^0 \cos s_2^0 - e^{i\alpha} \sin s_1^0 \sin s_2^0 \cos\beta \\ \sin s_1^0 \cos s_2^0 + e^{i\alpha} \cos s_1^0 \sin s_2^0 \cos\beta \\ \sin\beta \sin s_2^0 \end{pmatrix}, \tag{10}$$

with s_1^0, s_2^0, α and β arbitrary.

Since $|\psi^4\rangle = e^{i\varphi_g}|\psi^1\rangle$, the geometric phase for the complete circuit is extracted from the overlap real positive overlap $\langle\psi^{(3)}|\psi^{(1)}\rangle$. This works out immediately to

$$\varphi_g = \arg(\cos s_1^0 \cos s_2^0 - e^{-i\alpha} \sin s_1^0 \sin s_2^0 \cos\beta\,). \tag{11}$$

The generalization to SU(4) is immediate: the form of Eq.(7) remains the same, but the matrix of Eq.(8) is augmented to a 4×4 matrix:

$$R_{s_k} = \begin{pmatrix} \cos s_k & -\sin s_k & 0 & 0 \\ \sin s_k & \cos s_k & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

The condition of Eq.(9) remains. As we have argued, the first two vertices of the geodesic triangle remain unchanged, but the last vertex now depends on the 6 parameters of SU(4)/U(3):

$$\psi^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = e^{-i\varphi_g}\psi^{(4)}, \qquad \psi^{(2)} = \begin{pmatrix} \cos s_1^0 \\ \sin s_1^0 \\ 0 \\ 0 \end{pmatrix},$$

$$\psi^{(3)} = \begin{pmatrix} \cos s_1^0 \cos s_2^0 - e^{i\alpha} \sin_1^0 \sin s_2^0 \left(-\cos\beta_1 \cos\beta_2 + \sin\beta_1 \sin\beta_2 \cos\beta_3\right) \\ \sin s_1^0 \cos s_2^0 + e^{i\alpha} \cos s_1^0 \sin s_2^0 \left(\cos\beta_1 \cos\beta_2 - \sin\beta_1 \sin\beta_2 \cos\beta_3\right) \\ \left(\cos\beta_1 \sin\beta_2 + \sin\beta_1 \cos\beta_2 \cos\beta_3\right) \sin s_2^0 \\ \sin s_2^0 \sin\beta_1 \sin\beta_3 \end{pmatrix}, \tag{13}$$

a form which obviously reduces to the SU(3) case if $\beta_3 = 0$. For SU(4)/U(3), the Berry phase is again related to the inner product of $\langle\psi^{(1)}|\psi^{(3)}\rangle$ through $|\psi^{(4)}\rangle = e^{i\varphi_g}|\psi^{(1)}\rangle$ and can be seen to depend on the required number of parameters.

IV. GEOMETRIC PHASE IN SU(N) INTERFEROMETRY

An optical SU(N) transformation can be realized by a N–channel optical interferometer [10].

The $SU(2)_{12}$ matrix R_s in Eq. (8) is a special case of the generalized lossless beam splitter transformation for mixing channels 1 and 2. More generally a beam splitter can be described by a unitary transformation between two channels [8]. For example, a general $SU(2)_{23}$ beam splitter transformation for mixing channels 2 and 3 is of the form

$$R_{23}(\phi_{\mathrm{t}}, \theta, \phi_{\mathrm{r}}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_{\mathrm{t}}} \cos\theta & -e^{-i\phi_{\mathrm{r}}} \sin\theta \\ 0 & e^{i\phi_{\mathrm{r}}} \sin\theta & e^{-i\phi_{\mathrm{t}}} \cos\theta \end{pmatrix}, \tag{14}$$

with ϕ_{t} and ϕ_{r} the transmitted and reflected phase–shift parameters, respectively, and $\cos^2\theta$ the beam splitter transmission. A generalized beam splitter can be realized as a combination of phase shifters and 50/50 beam splitters in a Mach–Zehnder interferometer configuration.

The goal of the following is to construct an SU(3) optical transformations in terms of SU(2) elements which realize the geodesic evolution in the geometric space by appropriately adjusting parameters of the interferometer.

It will be convenient to write $\psi^{(3)}$ in Eq.(10) as $(e^{i\xi}\cos\eta, e^{i(\xi+\chi)}\sin\eta\cos\tau, \sin\eta\sin\tau)^t$, where ξ, η, τ and χ are functions of s_1^0, s_2^0, α and β, the parameters of $\psi^{(3)}$ in Eq. (10). Following our factorization scheme, the geodesic evolution operators $U_k^g(s_k)$, connecting $\psi^{(k)}$ to $\psi^{(k+1)}$, can be expressed as

$$\begin{aligned} U_1^g(s_1) &= R_{s_1}\,, \\ U_2^g(s_2) &= R_{s_1^0}\cdot R_{23}(\alpha,\beta,0)\cdot R_{s_2}\cdot R_{23}^{-1}(\alpha,\beta,0)\cdot R_{-s_1^0}\,, \\ U_3^g(s_3) &= R_{23}(\chi,\tau,\xi)\cdot R_{-s_3}\cdot R_{23}^{-1}(\chi,\tau,\xi)\,, \end{aligned} \tag{15}$$

with R_s given by Eq. (8), the parameters s_k ranging from $0\le s_k\le s_k^0$, and $s_3^0=\eta$. Note that s_3^0 and, in fact, all the parameters of $U_3^g(s_3)$ are fixed by the requirement that $\psi^{(4)}=e^{i\varphi_g}\psi^{(1)}$. Also note that, for each k, $U_k^g(0)$ is the identity in SU(3) and $U_k^g(s_k^0)\psi^{(k)}=\psi^{(k+1)}$ as required. Once it is observed that $\langle\,\psi^{(k+1)}|\;\psi^{(k)}\,\rangle=\cos s_k^0$, it is trivial to verify that each evolution satisfies Eq. (9) and is therefore geodesic.

The geometric phase for the cyclic evolution $\psi^{(1)}\to\psi^{(4)}$ is given explicitly by Eq.(11). This phase depends on four free parameters in the experimental scheme: s_1^0, s_2^0, α and β, which describe a general geodesic triangle in SU(3)/U(2).

The interferometer configuration for realizing the necessary evolution about the geodesic triangle is depicted in Fig. 1. This configuration consists of a sequence of $\mathrm{SU}(2)_{ij}$ transformations, and we use the shorthand notation $\Omega_i\equiv(\alpha_i,\beta_i,\gamma_i)$ to designate the three parameters associated with the generalized beam splitter. The three–channel interferometer consists of a sequence of nine $\mathrm{SU}(2)_{ij}$ transformations. The field enters port 1_{in}, and the vacuum state enters ports 2_{in} and 3_{in}.

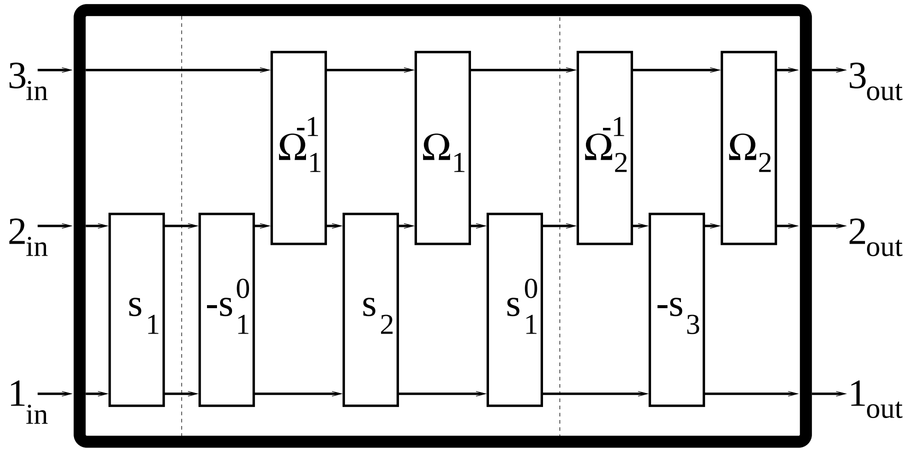

FIG. 1. The SU(3) interferometer is depicted, with three input ports 1_{in}, 2_{in} and 3_{in}, and three output ports 1_{out}, 2_{out} and 3_{out}. There are nine beam splitter transformations with parameters s_1, s_2, s_3, $\Omega_1 = (\alpha, \beta, 0)$, $\Omega_2 = (\chi, \tau, \xi)$, and $\Omega_i^{-1} = -\Omega_i$. For geodesic, cyclic evolution of the output state, only four parameters are independent.

For SU(4), the first evolution is the same, but the second will depend on more parameters as the dimensionality of SU(4)/U(3) is larger than that of SU(3)/U(2). Briefly, we have:

$$
\begin{aligned}
U_1^g(s_1) &= R_{s_1}\,, \\
U_2^g(s_2) &= R_{s_1^0} \cdot \tilde{V}_2 \cdot R_{s_2} \cdot \tilde{V}_2^{-1} \cdot R_{-s_1^0}\,, \\
U_3^g(s_3) &= V_3 \cdot R_{-s_3} \cdot V_3^{-1}\,,
\end{aligned}
\tag{16}
$$

where

$$
\tilde{V}_2 = R_{23}(\alpha_1, \beta_2, 0) \cdot R_{34}(\alpha_1, \beta_3, 0) \cdot R_{23}(0, \beta_1, 0)\,, \tag{17}
$$

and where V_3 is an SU(3) matrix of the form $R_{23} \cdot R_{34} \cdot R'_{23}$ whose details are unimportant for our purposes.

Although SU(3) and SU(4) interferometry have been considered in some details, the methods employed here can be extended to SU(N), or N–channel, interferometry [10]. The schemes discussed above employing such a device would produce and enable observation of the geometric phase shift for geodesic transformations of states invariant under U(N-1) subgroups of SU(N) states in the 2(N-1)–dimensional coset space SU(N)/U(N-1).

V. DISCUSSION AND CONCLUSION

This contribution is a summary of recent theoretical work on the possibility of measuring Berry phases using optical elements. The scheme depends on the optical realization of SU(N) transformations in the optical domain; this is possible because the Lie algebra su(n) can be realized in terms of boson creation and destruction operators which have immediate interpretation as photon field operators. There also exists the possibility of realizing Sp(2n,$\mathbb{R}$) transformation using optical elements [7]: the Lie algebra sp(2n,$\mathbb{R}$) also has a realization in terms of boson operators. The setup to measure Berry phase in an optical experiment is interesting because it provides a very practical realization of otherwise abstract ideas and allows one to do "experimental differential geometry" over SU(N)/U(N-1).

This contribution has dealt exclusively with the optical realization of Abelian Berry phase: even if states are invariant under U(N), two states are equivalent if they differ by a U(1)–phase. It is possible to enlarge the equivalence class to obtain the so–called non-Abelian Berry phase [13], which has been studied in the context of degenerate states. It is possible to study the non-Abelian version of the results presented here by using polarization: two states of different polarization are declared equivalent. The larger equivalence class comes about because a rotation of the polarization plane is an SU(2) transformation. The experimental aspects of this remain, at the moment, unclear. An optical experiment to measure an SU(2) phase would require optical devices which perform "tunable" polarization–dependent transformation. The theoretical aspects of this questions are currently under investigation.

This work has been supported by two Macquarie University Research Grants and by an Australian Research Council Large Grant. BCS appreciates valuable discussions with J. M. Dawes and A. Zeilinger, and HdG acknowledges the support of Fonds F.C.A.R. of the Québec Government.

REFERENCES

[1] Barry C. Sanders, Hubert de Guise, Stephen D. Bartlett and Weiping Zhang, to appear in Phys.Rev.Lett.

[2] M. V. Berry, Proc. Roy. Soc. (Lond.) **392**, 45 (1984).

[3] B. Simon, Phys.Rev.Lett **51**, 2167 (1983); F. Wilczek and A. Shapere, *Geometric Phases in Physics*, Advanced Series in Mathematical Physics - Vol. **5** (World Scientific, Singapore, 1989).

[4] A. Tomita and R. Y. Chiao, Phys.Rev.Lett **57**, 937 (1986); R. Y. Chiao, A. Antaramian, K. M. Ganga, H. Jiao, S. R. Wilkinson, and H. Nathel, Phys.Rev.Lett **60**, 1214 (1988); D. Suter, K. T. Mueller, and A. Pines, Phys.Rev.Lett **60**, 1218 (1988).

[5] P. G. Kwiat and R. Y. Chiao, Phys.Rev.Lett **66**, 588 (1991).

[6] R. Simon and N. Mukunda, Phys.Rev.Lett **70**, 880 (1993). J. Phys. A: Math. Gen. **32**, 7791 (1999).

[7] B. Yurke, S. L. McCall, and J. R. Klauder, Phys.Rev.A **33**, 4033 (1986).

[8] R. A. Campos, B. E. A. Saleh and M. C. Teich, Phys.Rev.A **40**, 1371 (1989).

[9] D. J. Rowe, B. C. Sanders and H. de Guise, J. Math. Phys. **40**, 3604 (1999).

[10] M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, Phys.Rev.Lett **73**, 58 (1994); B. C. Sanders, H. de Guise, D. J. Rowe and A. Mann,

[11] T. H. Chyba, L. J. Wang, L. Mandel, and R. Simon, Opt. Lett. **13**, 562 (1988); R. Simon, H. J. Kimble and E. C. G. Sudarshan, Phys.Rev.Lett **61**, 19 (1988).

[12] Arvind, K. S. Mallesh, and N. Mukunda,

[13] J. Anandan, Phys. Lett. A **133**, 171 (1988), F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).